Subspace-based 1-bit Wideband Spectrum Sensing

Junquan Deng*, Yong Chen

The Sixty-third Research Institute
National University of Defence Technology (NUDT)
Nanjing, China
jqdeng@nudt.edu.cn, cheny63s@nudt.edu.cn

WCSP 2019

This work focuses on

- Power-efficient wideband spectrum sensing for cognitive radio sensor networks

We consider

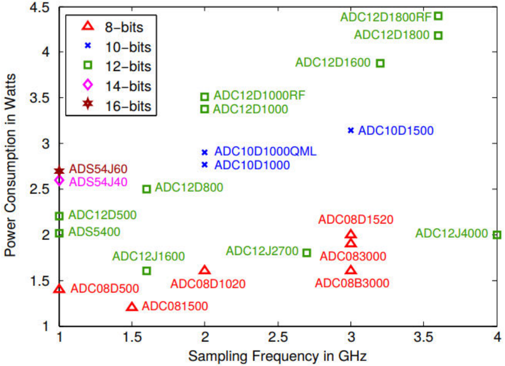
- Spectrum sensing in a wideband cognitive radio system where 1-bit ADCs are adopted at the RF sensors

The objective is

- To detect the occupation states of individual sub-bands simultaneously in a wide frequency range

High-speed high-resolution ADCs are expensive and power-hungry

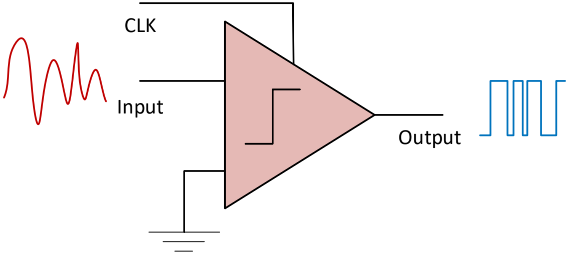
- The circuit complexity and the power consumption of a ADC grows exponentially with the sampling resolution[1]



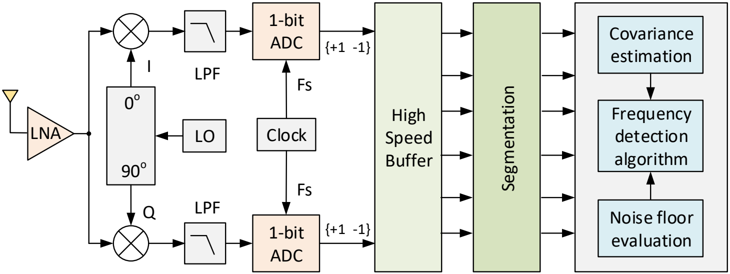
[1] B. Murmann, ADC Performance Survey, `http://web.stanford.edu/~murmann/adcsurvey.html`.

1-bit ADCs for wide-band spectrum sensing?

- Can be implemented using a single comparator
- Ultra-low driving power and circuit complexity
- Incurs only a small performance loss compared to high-resolution ADCs in low-SNR regime
- Have been considered for massive MIMO, low-cost radar

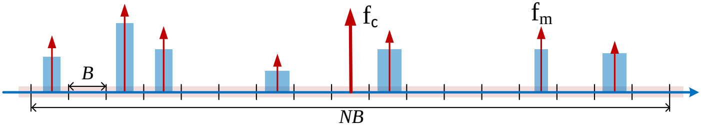


System architecture for 1-bit wideband spectrum sensing



- Homodyne RF architecture
- No automatic gain control (AGC) required
- Size of buffer can be greatly reduced
- Low signal processing complexity

1-bit Wideband Quantized Signal Model



Continuous analog signal:

$$y(t) = \sum_{m=1}^{M} \alpha_m(t) e^{\mathbf{j}2\pi f'_m (t-\tau_m)} + w(t), \qquad \text{(P1)}$$

Discrete received signal:

$$y[n] = \sum_{m=1}^{M} \alpha_m[n]\, e^{\mathbf{j}2\pi f'_m \left(\frac{n}{F_s} - \tau_m\right)} + w[n] \qquad \text{(P2)}$$

1-bit quantized signal:

$$q[n] = \frac{1}{\sqrt{2}} \left(\operatorname{sign}(\Re\{y[n]\}) + \mathbf{j}\operatorname{sign}(\Im\{y[n]\})\right) \qquad \text{(P3)}$$

Problem Formulation

- The M signals with frequencies $\{f_m\}_{m=1}^{M}$ are assumed to lie in exactly M sub-bands
- The objective of the RF sensor is to provide an N-bit digital word representing the states of the spectrum sub-bands
- We define $2N$ binary hypotheses $\{\mathcal{H}_{0,n}\}_{n=1}^{N}$ and $\{\mathcal{H}_{1,n}\}_{n=1}^{N}$, in which $\mathcal{H}_{0,n}$ denotes the idle state of the n-th sub-band and $\mathcal{H}_{1,n}$ represents the active state
- For each sub-band, a test statistics χ_n is formulated based on the 1-bit sampled data, and a test decision is given as follows:

$$\begin{cases} \text{Choose } \mathcal{H}_{0,n}, & \text{if } \chi_n < \theta_n, \\ \text{Choose } \mathcal{H}_{1,n}, & \text{if } \chi_n > \theta_n, \end{cases} \quad \text{for } n \in \{1, 2, \ldots, N\}, \tag{P4}$$

Subspace-based Technique for Wideband Spectrum Sensing

- Based on signal covariance, typical methods are MUSIC and ESPRIT

Received signals in vector form:

$$\mathbf{y} = \mathbf{s} + \mathbf{w} = [y[0], y[1], \cdots, y[N-1]]^{\mathrm{T}}, \tag{P5}$$

Covariance Matrix for $\mathbf{y}$:

$$\mathbf{R}_{\mathrm{yy}} = \mathbb{E}\left\{(\mathbf{s}+\mathbf{w})(\mathbf{s}+\mathbf{w})^{\mathrm{H}}\right\} = \mathbf{A}\boldsymbol{\Delta}\mathbf{A}^{\mathrm{H}} + \sigma_w^2\mathbf{I} \tag{P6}$$

- We have eigen-decomposition $\mathbf{R}_{\mathrm{yy}} = \mathbf{U}(\boldsymbol{\Lambda} + \sigma_w^2\mathbf{I})\mathbf{U}^{\mathrm{H}}$
- The signal and noise spaces are orthogonal for $\mathbf{R}_{\mathrm{yy}}$, we have

$$\mathbf{U} = [\mathbf{U}_{\mathrm{s}}\ \mathbf{U}_{\mathrm{n}}]$$

- $\mathbf{U}_{\mathrm{n}}$ of size $N \times (N-M)$ defines the noise subspace

Subspace-based Technique for Wideband Spectrum Sensing

The core idea is to estimate frequencies using the pseudo-spectrum

$$\mathrm{P}_{\mathrm{pseu}}(f) = \frac{1}{\mathbf{v}^{\mathrm{H}}(f)\mathbf{U}_{\mathrm{n}}\mathbf{U}_{\mathrm{n}}^{\mathrm{H}}\mathbf{v}(f)} = \frac{1}{\|\mathbf{U}_{\mathrm{n}}^{\mathrm{H}}\mathbf{v}(f)\|_2^2}. \quad \text{(P7)}$$

where $\mathbf{v}(f) = \left[1, e^{\frac{\mathbf{j}2\pi}{F_s}f}, e^{\frac{\mathbf{j}4\pi}{F_s}f}, \cdots, e^{\frac{\mathbf{j}2(N-1)\pi}{F_s}f}\right]^{\mathrm{T}}$ is the frequency-domain steering vector.

- If f equals one of the carrier frequencies of the spectrum components, the denominator is small, and there will be M largest peaks.

How to estimate the covariance based 1-bit quantized data?

- With 1-bit ADC, we only have $\mathbf{R}_{\mathrm{qq}} = \mathbb{E}\{\mathbf{q}\mathbf{q}^{\mathrm{H}}\}$
- According to Bussgang theorem and Vleck's arcsine law, we have

$$\mathbf{R}_{\mathrm{qq}} = \frac{2}{\pi}\left[\arcsin\left(\boldsymbol{\Sigma}_{\mathrm{y}}^{-\frac{1}{2}}\mathbf{R}_{\mathrm{yy}}\boldsymbol{\Sigma}_{\mathrm{y}}^{-\frac{1}{2}}\right)\right], \quad \text{(P8)}$$

where $\boldsymbol{\Sigma}_{\mathrm{y}} = \mathrm{diag}(\mathbf{R}_{\mathrm{yy}})$ and $\arcsin(\cdot)$ is element-wise.

- The normalized covariance for unquantized $\mathbf{y}$ can be approximated as

$$\bar{\mathbf{R}}_{\mathrm{yy}} \doteq \frac{\pi}{2}\mathbf{R}_{\mathrm{qq}} + \left(1 - \frac{\pi}{2}\right)\mathbf{I} \quad \text{(P9)}$$

- For the an eigenvector $\mathbf{v}$ of $\mathbf{R}_{\mathrm{yy}}$ with $\mathbf{R}_{\mathrm{yy}}\mathbf{v} = \lambda\mathbf{v}$, we have

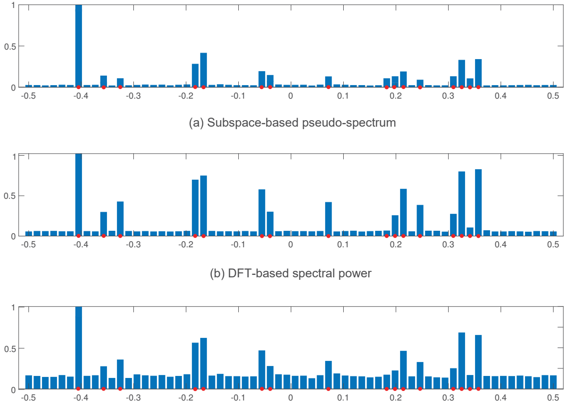
$$\frac{\pi}{2}\mathbf{R}_{\mathrm{qq}}\mathbf{v} \doteq \left(\frac{\lambda}{p} - 1 + \frac{\pi}{2}\right)\mathbf{v}, \quad \text{(P10)}$$

which implies that $\mathbf{R}_{\mathrm{qq}}$ and $\mathbf{R}_{\mathrm{yy}}$ have identical signal and noise spaces

Subspace-based 1-bit wideband spectrum sensing algorithm

1. Acquire L snapshots of 1-bit quantized data $\{\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_L\}$
2. $\hat{\mathbf{R}}_{\mathrm{qq}} \leftarrow \frac{1}{L}\sum_{l=1}^{L} \mathbf{q}_l \mathbf{q}_l^{\mathrm{H}}$, $\hat{\mathbf{R}}_{\mathrm{yy}} \leftarrow \frac{\pi}{2}\hat{\mathbf{R}}_{\mathrm{qq}} + \left(1 - \frac{\pi}{2}\right)\mathbf{I}$
3. $\hat{\mathbf{R}}_{\mathrm{yy}} = \hat{\mathbf{U}}\hat{\mathbf{\Lambda}}\hat{\mathbf{U}}^{\mathrm{H}}$, where $\hat{\mathbf{U}} = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_N]$, and $\hat{\mathbf{\Lambda}} = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_N\}$ with $\lambda_i \geq \lambda_j$ for $i < j$
4. Estimate the number of spectrum components using a Minimum Description Length(MDL) estimator
5. Partition $\hat{\mathbf{U}}$ into $[\mathbf{U}_{\mathrm{s}} \ \mathbf{U}_{\mathrm{n}}]$
6. Compute pseudo-spectrum $\frac{1}{\|\mathbf{U}_{\mathrm{n}}^{\mathrm{H}}\mathbf{v}(f)\|_2^2}$ for $f \in \{f_1, f_2, \ldots, f_N\}$
7. Find the $N - M$ smallest elements in pseudo-spectrum, estimate the noise floor $\mathrm{P}_{\mathrm{noise}}$ as the mean of the $N - M$ smallest elements
8. If $p_s(n) > 10^{\frac{\gamma}{10}}\mathrm{P}_{\mathrm{noise}}$ ($\gamma = 3$ dB), mark the n-th sub-band as occupied

Performance Evaluation



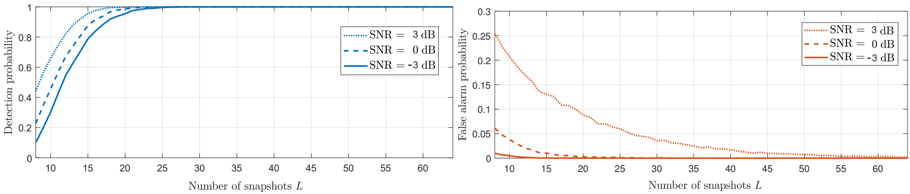
(a) Subspace-based pseudo-spectrum

(b) DFT-based spectral power

(c) Correlation-based spectral power

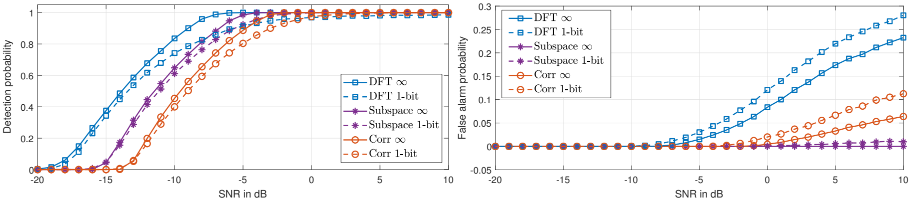
- Subspace-based method has a more distinguishable floor compared to FFT-based and correlation-based method

Time Resolution vs Detection Performances



- When SNR is 0, the proposed method has perfect performances with 32 snapshots, corresponds to a time-resolution of $3.2\ \mu s$
- When SNR is high, more snapshots of data are needed to attain a zero false alarm rate
- In high SNR regime, more samples are needed to average out the 1-bit quantization distortion in estimating the empirical covariance matrix
- 1-bit wideband spectrum sensing has a preferred operational SNR range

Performance Comparisons under Different SNR Conditions



- Performances with 1-bit ADCs are comparable to those with infinite-resolution ADCs
- The detection probability of the proposed method is lower than that of DFT-based and higher than correlation-based
- The proposed method achieves almost zero false alarm and is superior compared to the other two

Concluding Remarks

- We have proposed a subspace-based 1-bit wideband spectrum sensing method, it exhibits ultra-low power consumption, low memory and computation demands, and is suitable for larger-scale RF sensor network deployments.
- Our results suggest that the superiority of the subspace technique in parameter estimation translates into efficacy in 1-bit wideband spectrum sensing.
- We show by simulations that the proposed method exhibits near-zero false alarm while achieves similar detection probability as compared to other typical sensing methods.