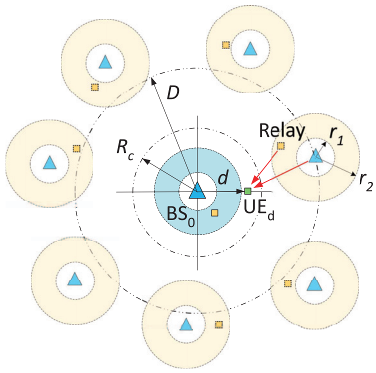


Fig. 2. A simplified model used in the analysis.

of the network, and devise advanced relaying management methods based on this characterization.

II. System Model

We consider a system where D2D relaying happens in multiple cells using the same channel. D2D relaying transmissions in neighboring cells will affect each other if they use the same radio resources, e.g. a common sub-channel. The system model considered is depicted in Fig. 1. There are multiple active user equipments (UEs) inside each cell. UEs in a cell are orthogonal, so that there is a transmission to a UE per sub-channel and there is no intra-cell interference. The UEs close to the BS will use direct one-hop DL transmission, while two-hop D2D relaying may be used for DL transmission to cell-edge UEs. Pathloss is modeled by two gain functions, $\ell_b(x)$ for BS-to-UE links and $\ell_d(x)$ for D2D links. Both are monotonically decreasing functions of distance x, with different pahtloss exponents η and η_d,

$$\ell_b(x) = K_b x^{-\eta} \text{ and } \ell_d(x) = K_d x^{-\eta_d}. \quad (1)$$

We assume that UEs are uniformly distributed in the cells and BSs are equipped with omnidirectional antennas. Without loss of generality, our analysis focuses on the cell with BS_0 located at the origin of the 2-D plane. We assume that all cells are similar. Each cell is modeled as a circular area with radius R_c. There is a ring of radius R_r $(0 < R_r < R_c)$, and UEs at distance larger than R_r would be served by D2D relaying. The D2D relays are assumed to reside in an annulus of inner radius r_1 and outer radius r_2. To simplify analysis, D2D relays are assumed to be uniformly distributed inside this annulus. There is a homogeneous network of other cells around the cell at origin. The transmit power of all BSs is P_b, and relays transmit with power P_d. We assume that there is a minimum distance D between the BS of interest and the neighboring BSs. To proceed with analysis, we assume that interfering BSs are uniformly distributed in the 2-D plane outside the circle of radius D with BS density ρ_{bs}. Fig. 2 demonstrates the network model used for analysis.

We assume that the D2D relay operates in a half-duplex mode. For an active UE with two-hop D2D relaying, the time resource is divided between the BS-to-relay (1st hop) and D2D (2nd hop) transmissions, according to the achievable rates on the two hops. The BS-to-relay link uses a fraction β of the time and the D2D link uses a fraction $1-\beta$. This factor will affect the interference spillage to neighboring cells.

III. Interference Characterization

A. DL Interference from Neighbor BSs without D2D relaying

We consider a UE_d located at distance d from the BS_0. When D2D relaying is not adopted, the interference to UE_d comes only from interfering BSs. We can use the fluid model proposed in [9] to estimate the aggregate DL interference for UE_d. As in Campbell's theorem [10], this fluid model transforms an expectation of a random sum over the discrete point process (PP) to an integral involving the PP intensity in the distribution area. In the fluid model, each area element $\mathrm{d}A$ contains $\rho_{bs}\mathrm{d}A$ BSs which contribute to the aggregate DL interference. We approximate the integration area by an annulus centred at UE_d, with inner radius as $D-d$ and outer radius ∞. Assuming the path loss exponent $\eta > 2$, the aggregate DL interference is derived using the method in [9]:

$$I_{dl}(d) = \frac{2\pi\rho_{bs}P_bK_b}{\eta-2}(D-d)^{2-\eta}. \quad (2)$$

B. Distribution of D2D Interference from One Neighbor Cell

To derive the interference distribution from D2D relaying, we first find the distance distribution between UE_d and the interfering D2D relay in a neighboring cell. As shown in Fig. 3, the interfering D2D relay is uniformly distributed in the annulus with inner radius r_1 and outer radius r_2. The probability density function (p.d.f) of the random distance X between UE_d and the interfering D2D relay can be calculated using the Crofton fixed points theorem [11]:

$$f_X(x|y,r_1,r_2) = \begin{cases} 0, & \text{if } x \in (0, y-r_2) \cap (y+r_2, \infty), \\ f_c(x|y,r_2), & \text{if } x \in [y-r_2, y-r_1] \cap [y+r_1, y+r_2], \\ \frac{r_2^2}{r_2^2-r_1^2}[f_c(x|y,r_2) - \frac{r_1^2}{r_2^2} f_c(x|y,r_1)], & \text{otherwise.} \end{cases} \quad (3)$$

Here, $f_c(x|y,r)$ is the p.d.f for the distance between a random point inside a circle (with radius r) and a fixed point (outside this circle) with distance y to the center of the circle,

$$f_c(x|y,r) = \frac{2x}{\pi r^2}\cos^{-1}\left(\frac{x^2+y^2-r^2}{2xy}\right) \approx \frac{2\sqrt{r^2-(y-x)^2}}{\pi r^2}, \quad (4)$$

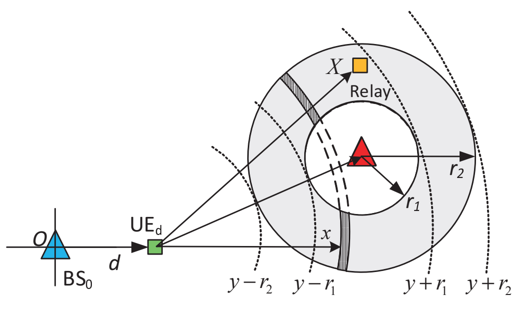


Fig. 3. Distance distribution for inter-cell D2D interference.

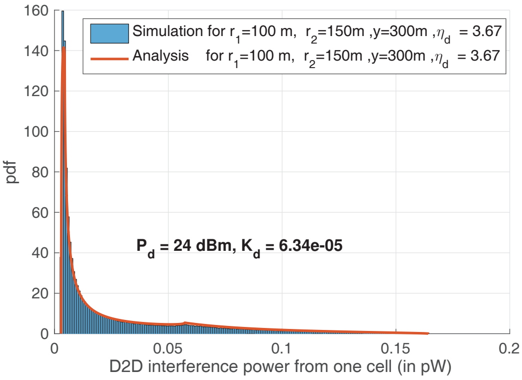


Fig. 4. Analysis and simulation for p.d.f of D2D interference.

with $y-r \leq x \leq y+r$. The interference power $I_d(x) = P_d\ell_d(x)$ is a monotonically decreasing function of distance x. The distribution function of D2D interference from one neighboring cell is

$$g_d(z|y, r_1, r_2) = -f_X(I_d^{-1}(z)|y, r_1, r_2)\frac{\partial I_d^{-1}(z)}{\partial z}. \quad (5)$$

Assuming the interference as a function of distance to be $I_d(x) = P_d K_d x^{-\eta_d}$, the inverse function is $I_d^{-1}(z) = (\frac{z}{P_d K_d})^{-1/\eta_d}$ and $\frac{\partial I_d^{-1}(z)}{\partial z} = \frac{-1/\eta_d}{P_d K_d}(\frac{y}{P_d K_d})^{-\frac{1}{\eta_d}-1}$. Fig. 4 shows the analytical p.d.f and the normalized histogram of a Monte Carlo simulation result for the D2D interference from one cell. The D2D interference distribution is long-tailed.

C. Distribution of aggregate D2D and DL interference

We assume that there is a probability p_r that a D2D relay is transmitting in a given interfering cell. This D2D relaying activity factor is determined by the relaying strategy, such as β, R_r, r_1 and r_2. The interference experienced by UE_d from a neighboring cell (either from a BS or a D2D relay) is a random variable

$$I(d, y_i) = (1 - X_i)I_b(y_i) + X_i I_d(y_i). \quad (6)$$

Here y_i is the distance from BS_i to UE_d, and $X_i \sim \mathbf{B}(1, p_r)$ is a Bernoulli random variable indicating whether neighboring cell i is using D2D relaying or not. If shadow fading is ignored we have a deterministic value $I_b(y_i) = P_b\ell(y_i)$, whereas $I_d(y_i) \sim g_d(z|y_i, r_1, r_2)$ is the D2D interference coming from the relay in cell i. The p.d.f of $I(d, y_i)$ is

$$f(x|d, y_i) = (1-p_r)\delta\left(x - P_b\ell(y_i)\right) + p_r g_d(x|y_i, r_1, r_2), \quad (7)$$

where $\delta(\cdot)$ is the Dirac delta function. The first moment of $I(d, y)$ is

$$\mathbb{E}[I(d, y)] = (1-p_r)P_b\ell(y) + p_r\mathbb{E}[I_d(y)], \quad (8)$$

where $\mathbb{E}[I_d(y)]$ is the average D2D interference from one cell,

$$\mathbb{E}[I_d(y)] = \frac{P_d K_d\left(\Phi(y, r_2) - \Phi(y, r_1)\right)}{r_2^2 - r_1^2} \quad (9)$$

with $\Phi(y, r) = r^2 y^{-\eta}\,{}_2F_1\left(\frac{\eta}{2}, \frac{1+\eta}{2}; 2; \frac{r^2}{y^2}\right)$, and ${}_2F_1(a, b; c; z)$ is the hypergeometric function.

If we consider the N_b closest interfering cells, then the total aggregate interference from these cells is:

$$I(d) \approx \sum_{i=1}^{N_b}(1-X_i)I_b(y_i) + \sum_{i=1}^{N_b} X_i I_d(y_i) = \sum_{i=1}^{N_b} I(d, y_i) \quad (10)$$

where $I(d, y_i) \sim f(x|d, y_i)$. $I(d)$ is a random variable dependent on p_r and the distribution of y_i, $i = 1, 2, ..., N_b$. The N_b distances are not independent since the interferences are experienced at a specific UE and the positions of the N_b neighboring BSs are spatially correlated if there is a minimum distance between interfering BSs. Different models for BS positions result in different joint distance distribution functions. Common models for BSs are the triangular lattice, square lattice, Poisson Point Process (PPP), Matérn hardcore point process etc. For analysis, we consider the network model depicted in Fig. 2 by assuming that the interfering BSs are uniformly and independently distributed outside the circle of radius D for the cell of interest. Using the fluid model, we have:

$$\mathbb{E}[I(d)] \approx (1-p_r)I_{dl}(d) + p_r\frac{M_d(D-d)^{2-\eta}(\phi(r_2) - \phi(r_1))}{(r_2^2 - r_1^2)(\eta - 2)} \quad (11)$$

where $\phi(r) = r^2\,{}_2F_1\left(\frac{\eta-2}{2}, \frac{\eta+1}{2}; 2; \frac{r^2}{(D-d)^2}\right)$ and the constant $M_d = 2\pi\rho_{bs}P_d K_d$. If there is no D2D relaying (with $p_r = 0$), the average interference is $I_{dl}(d)$. When D2D relaying is employed, some transmit power is offloaded from BS to D2D relays which are distributed around the BS. The offloaded power will cause random interference to UE receivers in neighboring cells. The p.d.f of $I(d)$ depends on several parameters including p_r, P_d, r_1 and r_2. When p_r, P_d and r_2 increase, the variance of $I(d)$ increases. The p.d.f of $I(d)$ is hard to derive since it involves an infinite number of convolutions over surface elements. For a given network of BSs, one can use the joint p.d.f for N_b closest BSs to find the distribution of $I(d)$. Numerical simulation results will be provided in section V.

IV. Interference-aware D2D relay selection and D2D resource allocation

To enable D2D relaying in cellular networks, D2D discovery, relay selection and resource allocation must be considered. We assume that UEs can perform channel measurements and report the channel information to the serving BS. A realtime controller located at the BS is responsible for relay selection and resource allocation for D2D relaying operations in its cell. We assume that a coordinated controller gathers information from the BSs on their interference measurements, and accordingly designs the relaying probabilities to be used in the network. The BS controller has the responsibility to keep the probability of relaying at the target value.

In this section, we discuss the details of the D2D relaying operation, including D2D channel spectral efficiency, relay selection and resource allocation. We assume that the BS controller has the knowledge of D2D channel pathloss for close UE pairs, and the aggregate interference from other BSs. We assume that the same DL D2D relaying strategy is applied in all cells, i.e. the same probability of D2D relaying, the same principle of relay selection and the same resource allocation scheme. The relaying decisions in one cell are made according to the current observed network state (e.g. aggregate interference for UEs and distribution of relay candidates in the cell). The aggregate interference for UE_d is determined by (11), and is close to $(1-p_r)I_{dl}(d)$ if transmit power of BS is much larger than the transmit power of D2D relays. In an interference-limited network, when D2D transmission probability p_r increases, the time-domain activity of BSs will decrease and the average aggregate interference will decrease since interference coming from BSs is larger than from relays on average.

A. Estimation of Optimal Relay Position for UE_d

To perform relay selection and resource allocation, an important task of the BS controller is to estimate the BS-to-relay and D2D spectral efficiencies. Assuming that $I_{dl}(d)$ for UE_d is known by the BS_0, the direct DL spectral efficiency for UE_d can be estimated from the Shannon formula and the fluid network model as

$$C_{dl}(d)=\log_2\left(1+\frac{P_b K_b d^{-\eta}}{\mathbb{E}[I(d)]+\sigma^2}\right), \tag{12}$$

where σ^2 is the noise power. Assuming that there is a large number of potential relay candidates, a cell-edge UE can always find a D2D relay which is close to the optimal relay position. In a circular homogeneous geometry, the best relay is on a radial line segment connecting the UE to the BS, at distance d_r from BS_0. With UE_d at d, we have $0 < d_r < d$. The estimated BS-to-relay spectral efficiency is $C_{dl}(d_r)$ and the D2D spectral efficiency between the optimal relay at d_r and UE_d is:

$$C_{d2d}(d,d_r)=\log_2\left(1+\frac{P_d K_d (d-d_r)^{-\eta_d}}{\mathbb{E}[I(d)]+\sigma^2}\right) \tag{13}$$

The maximum end-to-end two-hop spectral efficiency is determined by the harmonic mean of $C_{d2d}(d,d_r)$ and $C_{dl}(d_r)$, leading to the end-to-end throughput $C_{e2e}(d,d_r)=\beta C_{dl}(d_r)$ with the D2D activity factor

$$\beta=\frac{C_{d2d}(d,d_r)}{C_{dl}(d_r)+C_{d2d}(d,d_r)}. \tag{14}$$

For each value of d, there is an optimal relay position $d_{opt}(d)$ which maximizes the end-to-end throughput. If the system is interference-limited, we have

$$d_{opt}(d)=\arg\max_{0<d_r<d} C_{e2e}(d,d_r)\simeq\frac{2\eta_d-\ln(K)}{2\eta+2\eta_d}d, \tag{15}$$

where $K=\frac{P_d K_d D^{-\eta_d}}{P_b K_b D^{-\eta}}$.

B. Estimation of Optimal D2D Relay Transmission Probability

In a multi-cell network, D2D relaying decisions made in a cell (which determines the p_r in the cell) will affect the neighboring cell's decisions, and vice versa. The neighboring cells affect each other in an iterative way in a feedback loop driven by changes in aggregate interference, depending on p_r in each cell. In the equilibrium state of a homogeneous network, all cells should have the same D2D relaying probability p_r, and the same radiuses r_1 and r_2. With the optimal relay selection considered here, these numbers can be characterized in terms of a single radius R_r. A UE at distance d greater than R_r would use two-hop relaying. We assume that active UEs and D2D relay candidates are uniformly distributed in cells and that a cell edge UE with $d > R_r$ can always find a relay which is close to the optimal relay position for it. The distance of the optimal relay helping a UE at R_r would be r_1, and the distance of the relay helping a cell-edge UE at distance R_c would be r_2. The D2D activity factor p_r depends on the probability that the active UE is a in the relaying region ($d > R_r$), and the ratio $(1-\beta)$ of D2D transmission time. The relationship between D2D transmission probability p_r and R_r is:

$$\begin{aligned} p_r &= P(d>R_r)\mathbb{E}[1-\beta] \\ &\approx P(d>R_r)\mathbb{E}[\frac{C_{dl}(d_{opt}(d))}{C_{dl}(d_{opt}(d))+C_{d2d}(d,d_{opt}(d))}] \\ &= \frac{1}{2}\frac{R_c^2-R_r^2}{R_c^2}. \end{aligned} \tag{16}$$

In the equilibrium, R_r and p_r satisfy:

$$C_{dl}(R_r)=C_{e2e}(R_r,d_{opt}(R_r)) \tag{17}$$

There is no close form solution for R_r (and hence for p_r) constrained to (17) with general η. Given specific values of the network parameters, one can use a fixed-point method to search for R_r (and hence for p_r), as shown in Algorithm 1. After finding the equilibrium of R_r, we also get $\hat{r}_1=d_{opt}(R_r)$ and $\hat{r}_2=d_{opt}(R_c)$ which are the estimated inner and outer borders for the annulus where selected D2D relays are distributed.

Algorithm 1 Searching for R_r and p_r in equilibrium state

Initialization of R_r and p_r, using $R_r=R_c$ and $p_r=0$.
Calculate $C_\varepsilon=C_{dl}(R_r)-C_{e2e}(R_r,d_{opt}(R_r))$ using (12), (14).
while $|C_\varepsilon|>\epsilon$ **do**
  $R_r\leftarrow R_r+C_\varepsilon\Delta$, $p_r\leftarrow\frac{R_c^2-R_r^2}{2R_c^2}$.
  Calculate $C_\varepsilon=C_{dl}(R_r)-C_{e2e}(d,d_{opt}(R_r))$.
end while

V. Simulation Results

In this section, we consider two multi-cell networks for DL D2D relaying, i.e., a triangular lattice (hexagonal cells) and a square lattice (square cells). Detailed parameters are listed in Table I. Both networks have a minimum inter-BS distance $D=500\,m$ and the cell radius used in the analysis is $R_c=250\,m$.

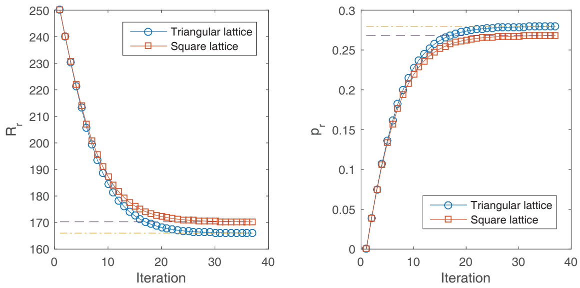
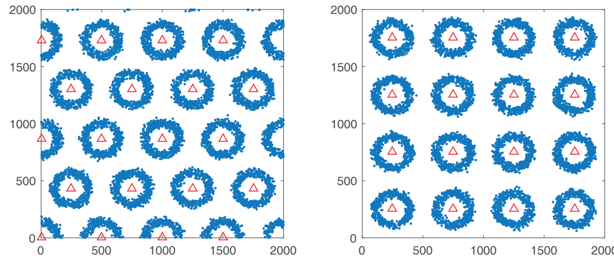


Fig. 5. R_r and p_r in equilibrium network state.



(a) Triangular lattice (b) Square lattice

Fig. 6. Distribution of selected D2D relays over multiple realizations.

A. Estimation of R_r and p_r in equilibrium state

Given the details of network parameters in Table I, R_r and p_r in equilibrium state can be estimated using Algorithm 1. An update step size $\Delta = 10$ and terminating condition $\epsilon = 10^{-3}$ bit/s/Hz are used. The search starts at $R_r = R_c$ with no D2D relaying allowed (i.e. $p_r = 0$). As R_r decreases, UEs at distance larger than R_r start to adopt two-hop D2D relaying. The average interference decreases as p_r increases. Fig. 5 shows that Algorithm 1 converges to $R_r = 166\,m$, $p_r = 0.28$ for the hexagonal network, and to $R_r = 170\,m$, $p_r = 0.27$ for the square cells. The estimated radiuses for the relay annulus are $\hat{r}_1 = 94\,m$, $\hat{r}_2 = 142\,m$ for hexagonal cells and $\hat{r}_1 = 96\,m$, $\hat{r}_2 = 142\,m$ for square cells.

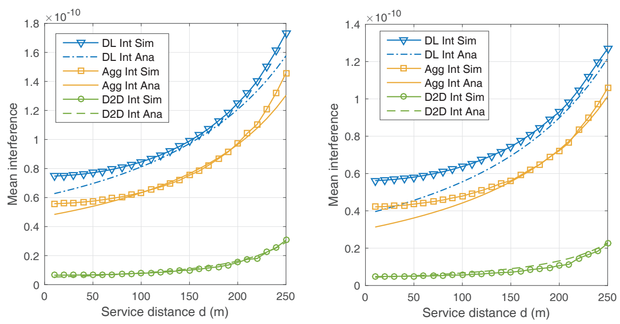
B. Analysis and Simulation Results on Aggregate Interference

We simulate the DL D2D relaying in the two network scenarios according to Table I. Relay selections and resource allocation for D2D relaying are based on the actual interference measurements. Simulations are carried out in an iterative way. There are $N_u = 10$ active UEs in each cell, TDMA and OFDMA are used to support multiple UEs. Each active UE in a cell obtains $1/N_u$ of radio resources to ensure proportional fairness. Each active UE gets one resource with a specific label. This resource is used either for direct transmission to the user, for transmission to the relay, or for D2D relaying transmission. Without interference coordination, each active UE will get a random resource. For each label, there is one active UE using this resource in a cell and this UE will be randomly distributed inside the cell. The transmissions (including DL and D2D) using the same resource in multiple cells would interfere each other. Each active UE located at distance larger than R_r from its serving BS would select the best relay from the relay candidates for its DL transmission. Fig. 6 shows the distribution of the selected D2D relays for 100 realizations of network instances, which justify the assumption of relay annulus. However, the actual outer annulus radiuses are larger than the estimated $\hat{r}_2$. The reason is that a cell edge UE can not always find a best relay close to the optimal relay position due to the finite density of relay candidates.

To understand the aggregate interference powers at a specific distance d from the serving BS, we generate 10 sample UE receivers randomly at each distance. Fig. 7 shows the average DL interference (without D2D relaying), total D2D interference and aggregate DL and D2D interference at different distances from the serving cell. Compared with the DL interference without D2D relaying, D2D interference is much smaller on average. The aggregate DL + D2D interference with D2D relaying is also smaller than the DL interference without D2D relaying. Fig. 8 shows the interference distribution for cell-edge UEs located at $d = 240\,m$ to the serving BSs in the hexagonal network. It shows that the major part of interference is from neighboring BSs. The probability that D2D interference dominates is quite small ($< 5\%$). The

TABLE I
NETWORK PARAMETERS.

Parameter	Symbol	Setting
System bandwidth	W	40 MHz
Minimum distance between BSs	D	500 m
Cell radius	R_c	250 m
BS density of triangular lattice	ρ_{bs}^h	$(\frac{\sqrt{3}}{2}D^2)^{-1}$
BS density of square lattice	ρ_{bs}^s	$(D^2)^{-1}$
Active UEs in each cell	N_u	10
Potential relays in each cell	N_r	100
PL model of cellular link	$\ell_b(x)$	$8.18 \times 10^{-5} x^{-3.67}$
PL model of D2D link	$\ell_d(x)$	$6.34 \times 10^{-5} x^{-3.67}$
TX power of BS	P_b	30 dBm
TX power of UE	P_d	24 dBm
Thermal noise power density	N_0	-174 dBm/Hz
Noise figure	N_f	8 dB



(a) Hexagonal, $R_r = 166\,m$ (b) Square, $R_r = 170\,m$

Fig. 7. Analytical and simulation results for mean interference.

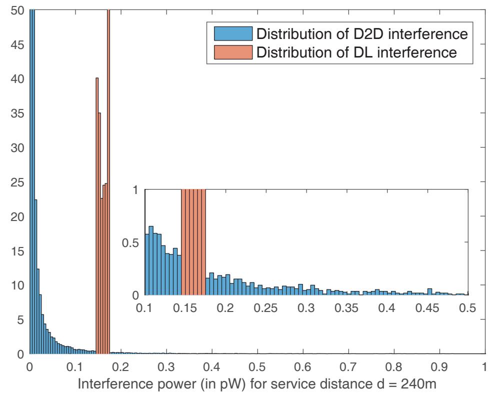


Fig. 8. Distribution of DL and D2D interferences for a UE at d = 240 m in the hexagonal network.

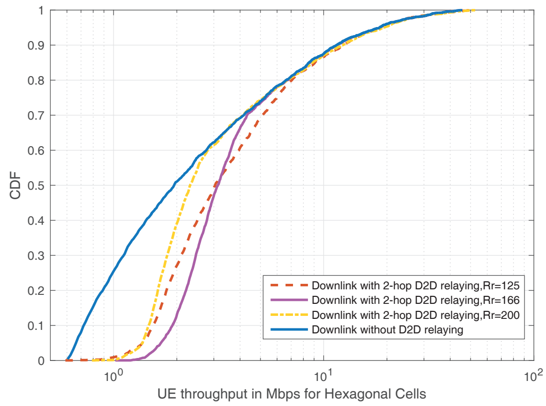


Fig. 9. CDF of UE DL throughput in hexagonal network.

aggregate DL interference distribution is concentrated while the aggregate D2D interference distribution is long-tailed.

C. Throughput performance of D2D relaying

Fig. 9 shows the CDFs of UE throughput performance for different D2D relaying strategies in the hexagonal network. The performances in a square network is similar, and thus is omitted. We use the optimal $R_r = 166\,m$ according to the analysis, and compare it with other values of R_r. D2D relaying helps to boost the performance of cell-edge UEs compared with the strategy that no D2D relaying is used. What's more, the setting of $R_r = 166\,m$ from the analytical equilibrium network state yields the best cell-edge performance compared to smaller or larger settings. For $R_r = 200\,m$, fewer cell-edge UEs adopts two-hop relaying, which increases the aggregate interference as the relaying probability p_r is smaller compared to $R_r = 166\,m$. For $R_r = 125\,m$, more UEs would use D2D relaying and the sum throughput for all active UEs is better than $R_r = 166\,m$. However, a portion of cell-edge UEs will be affected by the increasing inter-cell D2D interference, which makes the cell-edge performance lower than the strategy with $R_r = 166\,m$.

VI. Conclusion

We have considered two-hop D2D relaying to enhance end-to-end throughput in multi-cell downlink networks. We first study the characteristics of the aggregate co-channel interference when D2D relaying is applied in the networks. A fluid network model is used to analyze the inter-cell interference. To capture the complicated interference interaction in the multi-cell networks with D2D relaying, we have proposed an analytical model with several parameters, including a minimum relaying distance and a relaying probability. These parameters are optimized for network-level management. Simulation and analytical results are provided to understand the interference characteristic and throughput performance when D2D relaying is applied. The analytical model based on the fluid model provides a good approximation for the interference experienced in homogeneous networks with and without D2D relaying. The analytical optimization of network parameters, based on this model, works as a good tool for network-level D2D relaying management.

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